INTERNAL VELOCITY AND MASS DISTRIBUTIONS IN
CLUSTERS OF GALAXIES FOR A VARIETY OF COSMOGONIC MODELS

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1

ABSTRACT

The mass and velocity distributions in the outskirts $(0.5 - 3.0h^{-1}\mathrm{Mpc})$ of clusters of galaxies are examined for a suite of cosmogonic models utilizing large-scale Particle-Mesh (PM) simulations $(500^3$ cells, 250^3 particles and box size of $100h^{-1}\mathrm{Mpc}$, giving a nominal resolution of $0.2h^{-1}\mathrm{Mpc}$ with the true resolution about $0.5h^{-1}\mathrm{Mpc}$).

Through a series of model computations, designed to isolate the different effects, we find that both Ω_0 and P_k $(\lambda \leq 16h^{-1}\mathrm{Mpc})$ are important to the mass distributions in clusters of galaxies. There is a correlation between power, P_k, and density profiles of massive mass clusters; more power tends to point to the direction of a correlation between α and $M(r < 1.5h^{-1}\mathrm{Mpc})$ [see equation (1) for definitions], i.e., massive clusters being relatively extended and small mass clusters being relatively concentrated. A lower Ω_0 universe tends to produce relatively concentrated massive clusters and relatively extended small mass clusters compared to their counterparts in a higher Ω_0 model with the same power. Models with little (initial) small scale power, such as the HDM model, produce more extended mass distributions than the isothermal distribution for most of the clusters. But the CDM models show mass distributions of most of the clusters more concentrated than the isothermal distribution. X-ray and gravitational lensing observations are begining providing useful information on the mass distribution in and around clusters; some interesting constraints on Ω_0 and/or the (initial) power of the density fluctuations on scales $\lambda \leq 16h^{-1}\mathrm{Mpc}$ (where linear extrapolation is invalid) can be obtained when larger observational data sets, such as the planned Sloan Digital Sky Survey, become available.

With regard to the velocity distribution, we find two interesting points. First, in $0.5 < r < 3.0h^{-1}\mathrm{Mpc}$ region, velocity dispersions of four components, (1d, radial,

tangential, line-of-sight), show decreasing distributions as a function of cluster-centric distance in the three CDM models; but the HDM model shows just the opposite: weakly increasing velocity dispersions outwards. The CDM models can reasonably fit the observed galaxy velocity dispersions in the Coma cluster of galaxies but the HDM model provides a poor fit. Second, while the velocity dispersions among the three Cartesian directions are isotropic, a large scatter (40%) exists in all models. We find that for the scales $0.5 < r < 3.0h^{-1}$Mpc, the tangential velocity dispersion is always larger than the radial component by a factor of 1.2-1.6 in the CDM models and 1.3-2.0 in the HDM model. In all models the ratio of radial to tangential velocity dispersions is a decreasing function from $0.5h^{-1}$Mpc to $3.0h^{-1}$Mpc for massive clusters (smaller clusters tend to show a minimum for that ratio around $1.5 - 2.0h^{-1}$Mpc in the CDM models).

We also examine, in detail, the infall problem. Lower Ω_0 models are found to have larger turnaround radius for a fixed-mass clump than high Ω_0 models; this conclusion is insensitive to P_k. But we find that the following relation (between the turnaround radius, R_{ta}, and the mass within R_{ta}, M_{ta}), $\log_{10} R_{ta} = a + b \log_{10} M_{ta}$ ($a = -5.2 \pm 0.2$, $b = 0.40 \pm 0.02$, R_{ta} and M_{ta} are in h^{-1}Mpc and h^{-1}M$_\odot$, respectively), holds for all the models (the uncertainties in a and b indicate the variations among models). In addition, the relation between the overdensity inside the turnaround radius, δ_{ta}, and M_{ta} is fitted by $\log_{10} \delta_{ta} = c + d \log_{10} M_{ta}$ (*cf.* Table 1 for values of c and d). We show that the isolated spherical collapse model in an Einstein-de Sitter universe, having $\delta_{ta} = 9\pi^2/16 = 5.55$, gives a fair fit to results ($\sim 4 - 10$) of the nonlinear, non-spherical simulations performed here. Lower Ω_0 models have considerably higher δ_{ta}, $\sim 10 - 30$.

Finally, we find that the isothermal approximation (*cf.* equation 10) tends to underestimate the true masses within the Abell radius by 10-30% with a scatter of $\sim 50\%$ around the estimated mean (in the three hierachical models).

3

Subject headings: Cosmology: large-scale structure of Universe – cosmology: observations – cosmology: theory – dark matter – galaxies: clustering

1. INTRODUCTION

Since the dynamical time of clusters of galaxies is not much shorter than the Hubble time, it is expected that they contain useful information with regard to the early state of the universe. There are numerous studies from galactic scale to very large, supercluster scale in a variety of cosmological models and we will not even pretend to attempt to list them (in vain). For a recent review of confrontations of a panoply of cosmic theories with observations see Peebles & Silk (1988), for a post-COBE review of the Cold Dark Matter (CDM) model see Ostriker (1993), and for an extensive summary report on galaxy formation and large-scale structure see Silk & Wyse (1993). However, our current understanding of the evolution and properties of clusters of galaxies and of their relationships with details of cosmological models is still in its infancy. One is therefore encouraged to explore any new dimensions.

The central regions of clusters of galaxies are more relaxed than their outskirts and therefore are less sensitive to cosmological details. For example, the mass (within the Abell radius) function of clusters of galaxies is found to be dependent on the mean cosmological density, Ω_0, and the normalization on the relevant scale (e.g., σ_8), but not sensitively on the shape of the power spectrum within physical plausibility (Bahcall & Cen 1992; White, Efstathiou, & Frenk 1993). The outskirts of the clusters of galaxies ($1.5 - 3.0h^{-1}\mathrm{Mpc}$) are likely to be more sensitive to the details of a cosmological model than the central, core regions, since they have not undergone or have progressed relatively less toward virialization and an equilibrium state. One would therefore anticipate that the density and velocity fields in these regions ought to be dependent on such cosmological parameters as Ω_0, σ_8, P_k. No sufficiently detailed study on this subject has been done on the relevant scales. Motivated by this, this paper is written. Specifically, we will focus on the velocity and density fields surrounding the clusters of galaxies on the scales $0.5-3.0h^{-1}\mathrm{Mpc}$.

Traditionally, the velocity dispersions in clusters of galaxies are related to their masses. This interpretation is frequently utilized, and has become one of the two conventional ways to determine (based on dynamical grounds) masses of clusters using observed velocity information (Peebles 1970; Rood *et al.* 1972; White 1976; Kent & Gunn 1982; Merritt 1986; The & White 1986; Peebles 1993; Bahcall & Cen 1993). The alternative method is to use cluster X-ray temperature information (Sarazin 1986; Cowie, Henriksen, & Mushotzky 1987; The & White 1988; Hughes 1989; Henry & Arnaud 1991; Bahcall & Cen 1993), which is not the subject of this paper. The & White (1986) and Merritt (1986) found that the inferred mass of the Coma cluster of galaixes within a radius $1h^{-1}$Mpc is always very close to $6 \times 10^{14} h_{50}^{-1} M_{\odot}$, independent of details with regard to the possible variations of the velocity and mass distributions. However, the inferred mass within a radius of $2.7h^{-1}$Mpc is highly uncertain, ranging from $6 \times 10^{14} h_{50}^{-1} M_{\odot}$ to $5 \times 10^{15} h_{50}^{-1} M_{\odot}$. This large uncertainty is due to a large range of physically plausible configurations of the mass and galaxy velocity, which are consistent with the observed line-of-sight velocity dispersions of the galaxies in the Coma cluster.

In this paper we explore the cluster mass distribution directly using N-body simulations of a variety of cosmogonic models, and the velocity distribution under the assumption that the velocity field of galaxies in clusters follows that of the underlying mass, i.e., there is no velocity "bias". But note that the issue of the velocity "bias" is still a controversial one. Different results were found in the work by Carlberg, Couchman & Thomas (1990), Carlberg & Dubinski (1991), Cen & Ostriker (1992), Katz, Hernquist, & Weinberg (1992) and Evrard, Summers, & Davis (1994). But the uncertainty involved is being narrowed down, and at present the velocity bias value, $b_v \equiv v_{gal}/v_{mass}$, among different studies can be described by $b_v = 0.85 \pm 0.15$. Hence, although the previous assumption (there is no velocity bias) is not necessarily valid (but assumed for the sake of convenience of

comparison) and a definite conclusion awaits still higher resolution, larger scale, detailed, hydrodynamic computations with galaxy formation [for current, state-of-the-art work on this subject see Cen & Ostriker (1992, 1993a,b), Katz, Hernquist, & Weinberg (1992), and Evrard, Summers, & Davis (1994)], the effect is relatively small (at most 30%) even under the present uncertain situation.

In addition, we have to assume that the mass clumps in a simulation correspond to clusters of galaxies in the real universe. While one can not rule out the possibility that there exist massive, "dark" clusters in the real universe, which luminous galaxies happen to like to stay away from due to whatever physical processes, it seems unlikely that galaxies can resist enjoying the safe (deep) potential wells created by such massive clumps. One way to do this is to assume that these dark clusters are just formed and the luminous galaxies (which formed outside of them) have not had time to migrate in. But then one can not explain why such clusters are not X-ray luminous since the gas in the clusters should have been shock-heated during the phase of collapse to form the cluster. Based on these arguments, it seems inprobable that there exist any "dark" clusters in the real universe unless there is some large-scale process which segregates dark matter from baryons on scales larger than clusters at the early times. So we feel that it is a good assumption that mass traces clusters (either optical or X-ray), i.e., mass clumps in simulations correspond to clusters of galaxies with similar masses. However, we do not resolve dark halos with sizes smaller than $0.5h^{-1}$ under present simulations, and we do even worse to tag galaxies in the simulations. So let us stress once more that, it is an assumption not necessary a valid statement that galaxies spatially follow mass in the clusters, upon which our analyses are based.

We examine four different cosmogonic models: 1) standard COBE-normalized CDM model with $(h, \Omega_0, \sigma_8) = (0.5, 1.0, 1.05)$; 2) standard HDM model with $(h, \Omega_0, \sigma_8) =$ $(0.5, 1.0, 1.05)$; 3) an open CDM model with $(h, \Omega_0, \sigma_8) = (0.5, 0.2, 1.05)$; 4) an

open CDM model with $(h, \Omega_0, \sigma_8) = (0.5, 0.2, 1.05)$ but artificially adopting the $\Omega_0 = 1$ CDM initial power spectrum. The first model is a realistic, popular, COBE-normalized cosmogonic model. The remaining three models are not COBE-normalized (model 4 is not even physically realistic) but chosen to examine the parameter space (Ω_0, P_k) with the desire to isolate the different effects. The rest of the paper is organized in the following manner. Section 2 describes the numerical techniques; §3 presents the results; and §4 assembles our conclusions.

2. METHOD

2.1 Model Simulations

A standard Particle-Mesh code (PM, *cf.* Hockney & Eastwood 1981; Efstathiou *et al.* 1985) with a staggered-mesh scheme (see, e.g., Park 1990; Cen 1992) is used to simulate the evolution of the universal matter. We use $250^3 = 10^{7.2}$ particles on a 500^3 mesh with a periodic, comoving simulation box of size $100h^{-1}$Mpc, giving a nominal spatial resolution of $0.2h^{-1}$Mpc. The gravitational force is calculated by a FFT technique. The cloud-in-cell scheme is used to assign the gridded gravitational force to the disordered positions of particles as well as to calculate the gridded density from the disordered positions of particles. We denote a Hubble constant $H_0 = 100h$km/s/Mpc throughout.

Four models (listed in Table 1) are computed. Row 3 in Table 1 indicates the present mean density of the model universe in terms of the closure density; row 4 indicates the kind of power spectrum transfer function used; row 5 is the Hubble constant; row 6 is the power index of the spectrum on the large-scale end; row 7 is the mass fluctuation on a top-hat sphere with a radius $8h^{-1}$Mpc at present by normalizing the linear power spectrum; row 8 is simulation box size; row 9 is simulation cell size; row 10 indicates the mass of each particle in the simulation. The last four rows will be described in due course. We adopt the transfer functions

of Bardeen *et al.* (1986) for the CDM and HDM models. The initial density field for each simulation is generated assuming Gaussian fluctuations. The initial velocity field is given by the Zel'dovich approximation.

2.2 Cluster Identification

First, we select out clusters in a simulation using an adaptive friends-of-friends linking algorithm. The local linking length b_{ij} between the i-th and j-th particles is determined by $b_{ij} = \text{Min}[L_{\text{box}}/N^{1/3}, \beta(\frac{1}{2})^{1/3}(1/n_i(a_s)+1/n_j(a_s))^{1/3}]$, where L_{box} is the box size, N is the total number of particles in the box, $n_i(a_s)$ is the local number density at the i-th particle's position smoothed over a Gaussian window with radius a_s. We use $a_s = 10h^{-1}\text{Mpc}$ and $\beta = 0.25$, which are found to be adequate for our purpose in the sense that it produces neither too big structures (this will effectively reduce the number of centers of clusters if, for example, there are actually two big, adjacent structures which are artificially lumped together) nor too small structures (this might lead to treatment of small systems as rich clusters) compared to normal clusters of galaxies (see Bahcall & Cen 1992). Experiments with other linking schemes indicate that the selected list of clusters does not sensitively depend on the details of the selection scheme as long as there is a cutoff radius ($1.5h^{-1}\text{Mpc}$ in this case) and one counts all the particles within that radius. This yields a final list of clusters each with its center position, where the center of each cluster is the center of mass of each linked group. Such defined center of a cluster, in practice, always corresponds to the maximum of the central density peak of a cluster (except in rare cases if there are multiple dense structures in the central region, i.e., substantial substructures exist in the central region).

Second, having defined the centers of the clusters we return to the simulation box and count all the particles around each center to a certain radius (we only consider various properties within a radius of $3h^{-1}\text{Mpc}$ in this study). Typically, each

cluster contains hundreds to thousands of particles. Row 11 in Table 1 indicates the number of particles contained in a cluster of mass $1.0 \times 10^{14} h^{-1} M_\odot$. Also relevant is the number density of clusters in terms of mass (i.e., cluster mass function). As an illustration, we listed, as row 12 in Table 1, the cumulative number density of clusters with masses greater than $1.8 \times 10^{14} h^{-1} M_\odot$. We see that the standard, COBE-normalized, $\Omega_0 = 1$ CDM model overproduces such observed clusters by a factor in excess of ten. In contrast, the two lower Ω_0 CDM models agree well with observations. This subject concerning cluster mass function is very interesting by itself but we will not address this issue since it has been discussed in depth in Bahcall & Cen (1992).

Finally, we examine the properties of internal mass distribution and velocity distribution of each cluster as a function of radius [bin size (thichness of each shell) of $0.2 h^{-1}$Mpc is used, which is appropriate given our simulation resolution]. Other correlations among these and derived quantities are then also investigated in detail. In all cases, the mass of a certain region is defined to be the number of particles contained in that region multiplied by the mass of each particle. Note that since each shell ($0.2 h^{-1}$Mpc) contains typically few hundred or more particles, discreteness effect is small. For example, for an Abell cluster of mass $10^{14} h^{-1} M_\odot$, each shell will have $(740, 3703)$ particles (assuming the singular isothermal distribution for the simplicity of illustration) for $\Omega_0 = (1.0, 0.2)$ cases, which translate to Poissonian fluctuations of (3.7%,1.6%), respectively. Here we focus on the distributions as a function of radius (averaged over shells) but do not address the very important subject of substructures in and around clusters of galaxies. A subsequent paper will be devoted to this subject focusing on the dependence of substructures in and around clusters on P_k and Ω_0.

3. RESULTS

The main results are organized into three sections with regard to the mass distribution, velocity distribution, and the isothermal model.

3.1 Cluster Internal Mass Distribution

Figure (1) shows the density distribution (solid curves) as a function of cluster-centric distance for a few typical clusters in the four models [panel (a) for model 1, panel (b) for model 2, panel (c) for model 3 and panel (d) for model 4, see Table 1; this order will be maintained in the following figures]. Also shown as dashed lines are the slope for an isothermal sphere. Note that our nominal resolution (cell size) is $0.2h^{-1}$Mpc and the true resolution is about 2.5 cells. The density in the central region of each cluster, $r < 0.5h^{-1}$Mpc, is probably underestimated, due to our limited resolution. But this defect should not significantly affect the mass distribution on larger scales. We see that there is a wide range of density profiles within each model as well as among the different models. The three CDM models show a relatively close range of the asymptotic density slopes on scales $1 - 3h^{-1}$Mpc, the HDM model appears to have significantly shallower slopes. The CDM models have steeper slope than an isothermal case while the HDM model shows the opposite.

To address the mass distribution in a more quantitative fashion, we derive the asymptotic slope of the mass distribution around each cluster by fitting the simulated data points in the range $1.0 < r < 3.0h^{-1}$Mpc with a power law form as

$$M(< r) = Ar^\alpha \quad . \tag{1}$$

The relationship between α and $M(r < 1.5h^{-1}$Mpc$)$ is shown as open circles in Figure (2) for the four models. The solid line in each panel is the linear least-square fit for the open circles weighted by the inverse of the uncertainty of each power

law fit ($\Delta\alpha$). We see that, in the $\Omega_0 = 1$ CDM model [panel (a)], there is a weak trend of more massive clusters being more extended. In contrast, the HDM model shows a strong anti-correlation. Also note that there is a concentration around $\alpha = 2$ in the HDM model while it is near $\alpha = 0.3$ in the $\Omega_0 = 1$ CDM model. The primary reason for these differences ultimately traces to the fundamental differences of these two scenarios: the bottom-up picture in the CDM model and the top-down picture in the HDM model. The hierachical clustering process (or equivalently, the process of continuous merging) in the $\Omega_0 = 1$ CDM model tends to gradually make massive clusters more extended due to merging and infalling of satellite objects. On the contrary, the HDM model produces highly concentrated mass distribution in the regions where pancakes intersect; smaller clusters, which are the products of fragmentation of big clumps, filaments and sheets, tend to be more extended and lacking cores. The two open CDM models [panels (c,d)] appear to be intermediate between the above two models. Comparing panel (a) and panel (d) (note that the only difference between these two models is Ω_0, 1.0 vs. 0.2), we see that a lower Ω_0 has an effect of producing relatively less extended massive clusters. The obvious explanation for this is that there is significantly less merging in a lower Ω_0 universe than in an $\Omega_0 = 1$ universe. Comparing panel (c) and panel (d) [note that the only difference between these two models is the slope of the power spectrum on the relevant scales; model (3) (panel c) has a steeper slope of the power spectrum than model (4) (panel d)], we find, as a verification of our preceding explanation for the difference found between $\Omega_0 = 1$, CDM and HDM models, that less power (a steeper slope in model 3 than in model 4) on the relevant scales indeed points to the direction of an anti-correlation between α and $M(r < 1.5h^{-1}\mathrm{Mpc})$ (the HDM model is an extreme example of this). Also shown as a big star in each of the panels of Figure (2) is the data point for the Coma cluster of galaxies, where the Coma cluster mass within the Abell radius, $M(r < 1.5h^{-1}\mathrm{Mpc}) = 6.5 \times 10^{14}h^{-1}\mathrm{M}_\odot$, is

from the X-ray determination by Hughes (1989), and the asymptotic slope of the mass distribution in the Coma cluster ($\alpha = 0.27$) is adapted from The & White (1986). It is somewhat premature to make a definite conclusion based only on one data point of the Coma cluster, but if one were forced to choose among models, it seems that the two open models (panels c and d) fare well in producing Coma-like clusters, but the two flat models (panels a and b) appear to produce much more extended clusters for masses like that of the Coma. A reasonable, physically plausible amount of bias of galaxy distribution over mass on the relevant scales, will not significantly alter these remarks. A more realistic comparison requires to follow the galaxy motion as well as the dark matter motion in multi-component simulations, which at present are prohibitively expensive.

To explore this further, we show, in another way, the mass distribution in Figure (3), where the abscissa is the ratio of mass within a sphere of radius $1.5h^{-1}$Mpc to that within a sphere of radius $1.0h^{-1}$Mpc and the ordinate is the ratio of mass within a sphere of radius $3.0h^{-1}$Mpc to that within a sphere of radius $1.5h^{-1}$Mpc. Also shown as big solid dots are what one should have, if the density profile were isothermal. We find that most of clusters in all three CDM models tend to have mass distributions more concentrated than isothermal distribution (with some small fraction of clusters being exceptions). But the HDM model shows just the opposite, again due to the primary effect of fragmentation process in this model.

In summary, there are two factors which are important to the mass distributions. The first is the power (P_k) on relevant scales ($\lambda \leq 16h^{-1}$Mpc, assuming that the power on larger scales is the same); less power on the relevant scales tends to point to the direction of an anti-correlation between α and $M(r < 1.5h^{-1}$Mpc$)$, i.e., to make small mass clusters more extended and massive clusters more concentrated. The second factor is the mean density of the universe, Ω_0; less merging in a lower Ω_0 universe tends to make massive clusters more concentrated and less massive

13

ones more extended. CDM-like (hierachical) models produce density distributions of most of the clusters more concentrated than the isothermal distribution; on the contrary, HDM-like (pancaking) models produce density distributions of most of the clusters more extended than the isothermal distribution. The dependence on P_k and Ω_0 of the correlation between α and $M(r < 1.5h^{-1}\text{Mpc})$ as well the absolute amplitude of α might provide a way to decipher the initial power on the relevant scales ($\lambda \leq 16h^{-1}\text{Mpc}$) and/or Ω_0 of our universe, when more data on the cluster mass distributions becomes available.

3.2 Cluster Internal Velocity Distributions

We now turn to the velocity distributions. Figures (4,5,6,7) show the averages, $\langle \eta_m(r) \rangle$, of the four normalized velocity dispersions (as a function of cluster-centric distance) defined as

$$\eta_m(r) \equiv \sigma_m(r)/\langle \sigma_m \rangle \quad , \tag{2}$$

where σ_m is the velocity dispersion and $m = (1d, r, t, ||)$ for (1d, radial, tangential, line-of-sight), respectively. The one-dimensional velocity dispersion is defined as

$$\sigma_{1d} \equiv \frac{1}{\sqrt{3}}\sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_z^2} \quad . \tag{3}$$

The radial velocity dispsersion, σ_r, in a shell $r \to r+\Delta r$, is relative to the shell, i.e., the infall or outflow velocity of the shell is removed before calculating the velocity dispersion. The line-of-sight velocity dispersion in each cylindrical shell ($0.2h^{-1}\text{Mpc}$ thick, which cuts through the sphere of radius $3h^{-1}\text{Mpc}$) is computed by averaging over all the lines of sight in the shell. We define

$$\langle \sigma_m \rangle \equiv \frac{1}{R} \int_0^R \sigma_m(r)dr \quad , \tag{4}$$

where $R = 3h^{-1}\text{Mpc}$. The average, $\langle \eta_m(r) \rangle$, is over all the clusters with masses greater than $10^{13}h^{-1}M_\odot$ (solid line: number-weighted, dotted line: mass-weighted) in each model.

In Figure (4) we see that the three CDM models [panels (a,c,d)] have a decreasing 1-d velocity dispersion on scales from 1 to $3h^{-1}$Mpc. On the other hand, the HDM model [panel (b)] shows an increasing 1-d velocity dispersion with scale, which is consistent with its mass distribution (see §3.1). Figures (5,6) show the normalized radial and tangential velocity dispersions (solid line: number-weighted, dotted line: mass-weighted). We see similar results as in Figure (4).

Figure (7) shows the line-of-sight velocity dispersion as a function of projected distance. We see that the number-weighted line-of-sight velocity dispersions (solid lines) have weak, monotonically increasing distributions (in terms of projected cluster-centric distance). But the mass-weighted ones tend to tilt clockwise (i.e., to the direction of becoming decreasing functions in terms of scale). Let us take the observed line-of-sight velocity information on the Coma cluster of galaxies to make a comparison with these four models. Figure (8) shows the line-of-sight velocity dispersion averaged over the clusters with masses in the range from $3 \times 10^{14} h^{-1} \mathrm{M}_\odot$ to $1 \times 10^{15} h^{-1} \mathrm{M}_\odot$. Also shown are the observed data points (as solid dots) for the Coma cluster compiled from Table (2) of Kent and Gunn (1982). We see that the $\Omega_0 = 1$ CDM model (panel a) and $\Omega_0 = 0.2$ CDM model (panel c) provide good fits to observations from $r = 0.5h^{-1}$Mpc (below which our simulation is not reliable) to $3.0h^{-1}$Mpc. The reason that the panel (d) shows a lower velocity dispersion than that in panel (c) is that there are no clusters having mass greater than $4 \times 10^{14} h^{-1} \mathrm{M}_\odot$ in the simulated box of model 4 due to limited simulation boxsize. More quantitative comparison is not possible given both the uncertainty of the mass of the Coma cluster and the uncertainty of velocity bias of galaxies over matter. But it appears that the CDM-like models are viable. In contrast, the HDM model (panel b) provodes a poor fit noting that the shape of the computed velocity dispersion is very different from the observed counterpart.

Next, we study the issue of anisotropy of the velocity distributions. Let us first

define two measures for the anisotropy of the velocity distribution:

$$\epsilon_{||}(r) \equiv \sqrt{\frac{\sigma_{||,y}^2(r) + \sigma_{||,z}^2(r)}{2\sigma_{||,x}^2(r)}} \tag{5}$$

and

$$\epsilon_{rt}(r) \equiv \frac{\sigma_r(r)}{\sigma_t(r)} \quad . \tag{6}$$

The former measures the anisotropy among the three orthogonal Cartesian directions (x, y, z), and the latter measures anisotropy between the radial and tangential velocity dispersions (by a local observer). Figures (9,10) show $\epsilon_{||}(r)$ and $\epsilon_{rt}(r)$, respectively, as a function of projected cluster-centric distance, r_p, and 3-d cluster-centric distance, r, respectively, averaged over all the clusters with masses greater than $10^{13}h^{-1}M_\odot$ (equally weighted). We see, in Figure (9), that all the models have a mean of $\epsilon_{||}$ around unity, i.e., isotropic, but they show quite significant amount of scatters around the mean, 40-50%. Examination of Figure (10) (solid lines: number-weighted, dotted lines: mass-weighted) reveals that the velocity dispersions in all models are anisotropic between radial and tangential components. It shows that the tangential velocity dispersions are larger than the corresponding radial velocity dispersion by a factor of 1.2-1.6 in the CDM models and 1.3-2.0 in the HDM model. In all models the ratio of radial to tangential velocity dispersions show a decrease from $0.5h^{-1}$Mpc to $3.0h^{-1}$Mpc for massive clusters (smaller clusters tend to show a minimum for that ratio around $1.5 - 2.0h^{-1}$Mpc in the CDM models).

Let us now turn to the infall issue. Figure (11) shows the radial velocity of each cluster-centric shell relative to an observer at the cluster center as a function of cluster-centric distance, averaged over all the clusters with $M(r < 1.5h^{-1}\text{Mpc}) > 10^{13}h^{-1}M_\odot$ (solid line: number-weighted, dotted line: mass-weighted). Note that in a homogeneous uniform universe $v_r = Hr$. We see, as expected, that flat $(\Omega_0 = 1)$ models have larger turnaround radii (R_{ta}), where $v_r = 0$, than lower Ω_0 models,

16

and massive clusters have larger turnaround radii than poorer clusters. Figure (12) shows R_{ta} as a function of the mass inside the turnaround radius, M_{ta} (solid dots). Also shown as the solid lines are the least-square fitting curves with the following formula:

$$\log_{10} R_{ta} = a + b \log_{10} M_{ta} \quad , \tag{7}$$

where R_{ta} is in $h^{-1}\mathrm{Mpc}$ and M_{ta} is in $h^{-1}\mathrm{M_\odot}$. The values of a and b of the four fits are tabulated as row 13 in Table 1. It is interesting that in all the four models the values of a's and b's are close and can be represented by

$$a = -5.2 \pm 0.2 \quad \text{and} \quad b = 0.40 \pm 0.02 \quad . \tag{8}$$

It is very instructive to look closely at the three CDM models. We see that panels (c) and (d) look very similar [note that the only difference between these two models is the power P_k: panel (d) has more power than panel (c)]. This means that plausible variations of P_k have negligible effect on the turnaround radius for a given cluster mass. However, we notice that the points in panel (a) are always below those in panel (c) [and (d)] for a fixed mass, which seems surprising at first. The reason is the following. In order to accumulate the initially uniformly distributed matter into islands of matter, larger spatial volumes of matter are required in a lower Ω_0 model than in a higher Ω_0 model to reach the same masses. The countervailing factor is that a lower Ω_0 tends to brake the building-up process, thus reduces the infall. But in the models we examine here, $\Omega_0 = 1$ versus $\Omega_0 = 0.2$ models, it seems that the former factor is primary. Note that, if the former factor were the only one, the ratio of the two turnaround radii of panel (a) to panel (c) at a fixed mass should be $(0.2/1.0)^{1/3} = 0.58$ whereas we find the actual ratio is 0.79. We would like to stress that, at a fixed cluster-centric distance or at a fixed overdensity, high Ω_0 models have larger turnaround radii than low Ω_0 models (for a similar normalization, e.g.,

17

σ_8), as was shown in Figure (11); this topic has been discussed in Cen (1994) focusing on the kinematic behaviors of the Local Supercluster. Another related topic has been discussed by Peebles *et al.* (1989) with regard to the Local Group dynamics, concluding that a flat model is consistent with observed motions in the local group. We note, however, that the $\Omega_0 = 1$ model which fits the observed infall motion in the Local Group (our Galaxy relative to the Andromeda Nebula) requires $H = 80$km/s/Mpc, as found by Peebles *et al.* (1989), giving the age of the universe of 8.2 billion years, which seems too short. If one presses hard on the age issue, a lower Ω_0 model might provide a better fit, since a lower Ω_0 model will have a smaller infall motion at a fixed separation (the Galaxy relative to the Andromeda), which seems to point to the right direction of having a longer age. Very high resolution simulations, taking into account of large scale environmental effects, are needed before we can make more quantitative assessments of what ranges of Ω_0 and H_0 fit.

Finally, we look at the relation between mass overdensity inside turnaround radius, δ_{ta}, and M_{ta}. Figure (13) shows δ_{ta} as a function of M_{ta}. We see that the flat models have a lower overdensity than the open models ($\sim 4 - 10$ vs. $\sim 10 - 30$) within the turnaround radius. Also shown as the horizonal, dashed lines (in panels a and b) are the analytic prediction for an isolated spherical collapse case in an Einstein-de Sitter universe (*cf.* Peebles 1980). It is interesting that this analytic calculation gives a reasonable fit to the fully nonlinear, non-spherical simulations (panels a and b). The three CDM models all show a trend that massive clumps are likely to have lower values of overdensity inside the turnaround spheres than small mass clumps. We fit the open circles in panels (a,c,d) by the following formula:

$$\log_{10} \delta_{ta} = c + d \log_{10} M_{ta} \quad , \tag{9}$$

where M_{ta} is in $h^{-1} M_\odot$. The values of c and d are listed as row 14 in Table 1 (we do not fit for the HDM model).

3.3 The Isothermal Model

It is frequently assumed that the cluster density distribution can be approximated by an isothermal profile (Peebles 1993). This leads to a way to estimate the cluster mass given its velocity information as follows.

$$M_{ISO}(< r) = \frac{2v_{1d}^2(< r)r}{G} \quad , \tag{10}$$

where $v_{1d}(< r)$ is the 1-d velocity dispersion within a sphere of radius r; G is the gravitational constant. We now examine this issue in numerical simulations assuming that observed velocity information is error-free. Figures (14,15,16) show the ratios of the derived masses by assuming an isothermal distribution [equation (8)] to the true masses with spheres of radii $1.0h^{-1}$Mpc, $1.5h^{-1}$Mpc and $3.0h^{-1}$Mpc, respectively. We see that, in all three scales for all the models, there is a rather significant scatter of estimated masses. More seriously, with a radius $r = 1h^{-1}$Mpc, the isothermal approximation seems to underestimate the mean by 20-30% in the CDM models, although the scatter is still large so in some case they agree within scatter. On the scale $r = 1.5h^{-1}$Mpc, the isothermal approximation still underestimates the true masses by 10-20% in the CDM models but the agreement is better especially for poorer clusters ($M < 10^{14}h^{-1}M_\odot$). Then on the scale $r = 3.0h^{-1}$Mpc, the isothermal approximation overestimates the true masses by 10-30% in the CDM models. The HDM model shows that the isothermal model typically overestimates the masses for scales $r < 1.5h^{-1}$Mpc but underestimates the masses for scales $r > 1.5h^{-1}$Mpc. It seems that there is a scale around $2h^{-1}$Mpc where, accidentally, one may be able to get the right mass (on average) using the isothermal approximation.

Combining this information with the anisotropy found in Figure (9) ($\epsilon_{||}$), we conclude that the isothermal approximation tends to underestimate the mean of

the true masses within the Abell radius by 10-30% with a scatter of $\sim 50\%$ around the estimated mean (in the three hierachical models).

4. CONCLUSIONS

Our main conclusions can be summarized as four points with regard to the mass distribution, velocity distribution, infall motion, and the isothermal model.

(1) We find, by isolating the effects of Ω_0 and P_k (on the relevant scales, $\lambda \leq 16h^{-1}$Mpc) through a series of model simulations, that both Ω_0 and P_k are important to the mass distributions in clusters of galaxies. In the present study, we focus on the mass distributions in the outskirts of clusters ($r = 0.5 - 3h^{-1}$Mpc). Our main conclusion on this issue is assembled through three related points. First, there is a correlation between power, P_k (on the relevant scales), and density profiles of massive mass clusters; more power tends to point to the direction of a correlation between α and $M(r < 1.5h^{-1}$Mpc) [see equation (1) for definitions], i.e., massive clusters being more extended and small mass clusters being relatively concentrated. Second, a lower Ω_0 universe tends to produce relatively concentrated massive clusters and relatively extended small mass clusters compared to their counterparts in a higher Ω_0 model with the same power. Third, models with little (initial) small scale power, such as the HDM model, tend to produce more extended mass distribution for most of the clusters than the isothermal distribution. But the CDM-like models show mass distributions of most of the clusters more concentrated than the isothermal distribution.

X-ray observations, such as ROSAT and future satellite missions, and observations of gravitational lensing of distant galaxies by foreground clusters, producing effects such as coherent ellipticity (Miralda-Escude 1991; Blandford *et al.* 1991; Kaiser 1992), may provide useful information on the mass distribution in and around clus-

ters. In fact, they are providing us with some new, interesting observational results; see Bonnet *et al.* (1994), Mellier *et al.* (1994), Fahlman *et al.* (1994), Tyson (1994), Dahle *et al.* (1994), Smail *et al.* (1994) for the lastest observational work on this subject. Comparison between observations and detailed model computations could yield some interesting constraints on the (initial) power of the density fluctuations on scales $\lambda \leq 16h^{-1}\mathrm{Mpc}$ (where linear extrapolation is invalid) and/or Ω_0.

(2) With regard to the velocity distribution, we divide our conclusion into two points. First, in $0.5 < r < 3.0h^{-1}\mathrm{Mpc}$ region, velocity dispersions of four components, (1d,radial, tangential,$\|$), show decreasing distributions as a function of cluster-centric distance in the three CDM models; but the HDM models shows just the opposite: weakly increasing velocity dispersions outwards. The CDM models can reasonably fit the observed galaxy velocity dispersions in the Coma cluster of galaxies but the HDM provides a poor fit. Second, while the velocity dispersions among the three Cartesian directions are isotropic, a large scatter (40%) exists in all models. We find that for the scales $0.5 < r < 3.0h^{-1}\mathrm{Mpc}$, the tangential velocity dispersion is always larger than the radial component by a factor of 1.2-1.6 in the CDM models and 1.3-2.0 in the HDM model. In all models the ratio of radial to tangential velocity dispersions show a decrease from $0.5h^{-1}\mathrm{Mpc}$ to $3.0h^{-1}\mathrm{Mpc}$ for massive clusters (smaller clusters tend to show a minimum for that ratio around $1.5 - 2.0h^{-1}\mathrm{Mpc}$ in the CDM models).

(3) The relation between the turnaround radius and the mass within that radius can be approximated by $\log_{10} R_{ta} = a + b \log_{10} M_{ta}$ where $a = -5.2 \pm 0.2$, $b = 0.40 \pm 0.02$, R_{ta} is in $h^{-1}\mathrm{Mpc}$ and M_{ta} is in $h^{-1}\mathrm{M}_\odot$ (valid for all the models examined here). Lower Ω_0 models are found to have larger turnaround radius for a fixed mass clump than high Ω_0 models; this conclusion is insensitive to P_k. The relation between the overdensity inside the turnaround radius and the mass within that radius is fitted by $\log_{10} \delta_{ta} = c + d \log_{10} M_{ta}$ and values of c and d are listed as

the last row in Table 1. We show that the isolated spherical collapse model in an Einstein-de Sitter universe, having $\delta_{ta} = 9\pi^2/16 = 5.55$, gives a fair fit to results ($\sim 4 - 10$) of the nonlinear, non-spherical simulations performed here (see Figure 13). Lower Ω_0 models have considerably higher δ_{ta}, $\sim 10 - 30$. All models show a trend that massive clumps have lower values of overdensity than small mass clumps.

(4) The isothermal approximation (*cf.* equation 10) tends to underestimate the true masses within the Abell radius by 10-30% with a scatter of $\sim 50\%$ around the estimated mean (in the three hierachical models). Accidentally, it seems that within a sphere of radius $\sim 2h^{-1}$Mpc, one may be able to get the right mass (on average, in all the models) using the isothermal approximation but the scatter around the mean still exists.

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FIGURE CAPTIONS

Fig. 1– The density distribution as a function of cluster-centric distance for a few typical (randomly chosen) clusters in the four models [panel (a) for model 1, panel (b) for model 2, panel (c) for model 3, panel (d) for model 4, see Table 1; this order will be maintained in the subsequent figures]. The dashed lines indicate the case for an isothermal sphere.

Fig. 2– The relationship between α and $M(r < 1.5h^{-1}\mathrm{Mpc})$ [$cf.$ equation (1) for definitions] is shown as open circles. The solid line in each panel is the linear least-square fit for the open circles weighted by the inverse of the uncertainty of each power law fit ($\Delta\alpha$). Also shown as a big star in each of the panels is the data point for the Coma cluster of galaxies, where the Coma cluster mass within the Abell radius ($M(r < 1.5h^{-1}Mpc) = 6.5\times10^{14}h^{-1}M_\odot$) is from X-ray determination by Hughes (1989) and the asymptotic slope of the mass distribution in the Coma cluster ($\alpha = 0.27$) from The & White (1986).

Fig. 3– The abscissa is the ratio of mass within a sphere of radius $1.5h^{-1}\mathrm{Mpc}$ to that within a sphere of radius $1.0h^{-1}\mathrm{Mpc}$ and the ordinate is the ratio of mass within a sphere of radius $3.0h^{-1}\mathrm{Mpc}$ to that within a sphere of radius $1.5h^{-1}\mathrm{Mpc}$. Also shown as big solid dots are what one should have if the density profile is isothermal.

Fig. 4– The average of the normalized 1-d velocity dispersions as a function of cluster-centric distance [see equations (2,3,4) for definitions, §3.2].

Fig. 5– The average of the normalized radial velocity dispersions as a function of cluster-centric distance [see equations (2,4) for definitions, §3.2].

Fig. 6– The average of the normalized tangential velocity dispersions as a func-

26

tion of cluster-centric distance [see equations (2,4) for definitions, §3.2].

Fig. 7– The average of the normalized line-of-sight velocity dispersions as a function of projected cluster-centric distance [see equations (2,4) for definitions, §3.2].

Fig. 8– The line-of-sight velocity dispersions as a function of projected cluster-centric distance for the four models, averaged over clusters with masses in the range from $3 \times 10^{14} h^{-1} M_\odot$ to $1 \times 10^{15} h^{-1} M_\odot$. Also shown as solid dots are the data points for the Coma cluster taken from Table 2 of Kent & Gunn (1982).

Fig. 9– The velocity anisotropy measure, $\epsilon_{||}$ [see equation (5) for definition], among the Cartesian orthogonal directions, as a function of cluster-centric distance, r, averaged over all the clusters with masses greater than $10^{13} h^{-1} M_\odot$. ∎

Fig. 10– The radial and tangential velocity anisotropy measure, ϵ_{rt} [see equation (6) for definition], as a function of cluster-centric distance, averaged over all the clusters with mass greater than $10^{13} h^{-1} M_\odot$ (solid curves: number-weighted, dotted curves: mass-weighted).

Fig. 11– The radial velocity (as observed by an observer sitting at the center of the cluster) as a function of cluster-centric distance, averaged over all the clusters with mass greater than $10^{13} h^{-1} M_\odot$ (solid curves: number-weighted, dotted curves: mass-weighted).

Fig. 12– The turnaround radius, R_{ta}, as a function of the mass within that radius, M_{ta} (solid dots). Also shown as solid lines are the least-square fits [cf. equations (7,8) and Table 1].

Fig. 13– The overdensity inside the turnaround radius, δ_{ta}, as a function of the mass within that radius, M_{ta} (open circles). The dashed, horizontal lines in panels (a,b) are the result for a nonlinear, isolated spherical collapse

27

model in an Einstein-de Sitter universe (5.55). Also shown as solid lines are the least-square [*cf.* equation (9) and Table 1].

Fig. 14– The ratio of the derived mass by assuming an isothermal distribution [equation (10)] to the actual computed mass with spheres of radius $1.0h^{-1}$Mpc as a function of the true mass.

Fig. 15– The ratio of the derived mass by assuming an isothermal distribution [equation (10)] to the actual computed mass with spheres of radius $1.5h^{-1}$Mpc as a function of the true mass.

Fig. 16– The ratio of the derived mass by assuming an isothermal distribution [equation (10)] to the actual computed mass with spheres of radius $3.0h^{-1}$Mpc as a function of the true mass.

Table 1. Summary of the computed models

Row	Run	1	2	3	4
2	Model	CDM	HDM	CDM	CDM
3	Ω_0	1	1	0.2	0.2
4	TF	CDM $\Omega_0 = 1$	HDM $\Omega_0 = 1$	CDM $\Omega_0 = 0.2$	CDM $\Omega_0 = 1.0$
5	h	0.5	0.5	0.5	0.5
6	n	1	1	1	1
7	σ_8	1.05	1.05	1.05	1.05
8	$L(h^{-1}\mathrm{Mpc})$	100	100	100	100
9	$\Delta l(h^{-1}\mathrm{Mpc})$	0.2	0.2	0.2	0.2
10	$m_p(h^{-1}M_\odot)$	1.8×10^{10}	1.8×10^{10}	3.5×10^{9}	3.5×10^{9}
11	N_p^*	5555	5555	27777	27777
12	n_{model}/n_{obs}^{**}	$19.0 \pm 1.4/1.5^{+0.75}_{0.5}$	$5.0 \pm 0.7/1.5^{+0.75}_{0.5}$	$0.9 \pm 0.3/1.5^{+0.75}_{0.5}$	$1.9 \pm 0.4/1.5^{+0.75}_{0.5}$
13	a/b	$-5.3/0.40$	$-5.0/0.38$	$-5.2/0.40$	$-5.2/0.41$
14	c/d	$4.4/-0.26$	—	$4.2/-0.21$	$4.0/-0.20$

* number of particles contained in a cluster with mass $1.0 \times 10^{14}h^{-1}M_\odot$.

** cumulative number density of clusters with masses (with Abell radius) greater than $1.8 \times 10^{14}h^{-1}M_\odot$ for the model and observation. The one σ error bars for models are Poissonian and the observation is from Bahcall & Cen (1993).